Final Report

Base 10 Materials in Mathematics:
Legibility Study

Robert D. Glass

American Printing House for the Blind

Louisville, Kentucky

1983

Pamphlet File
Research [illegible]

Running Head: Base 10 Materials in Mathematics

Introduction

Participants in the 1979 American Printing House (APH) meeting on needs of blind students in mathematics established as the highest priority the development of entry level, primary grade materials which are designed to introduce and teach concretely basic math concepts and operations to blind students. High priority was given to the adaptation and evaluation of commercially available Base 10 materials (often referred to as exponential blocks or Number-Blox^{TM}) which can be used to introduce a wide variety of mathematical concepts at the entry level. These materials were also recommended by experts assessing mathematical content for APH and have been repeatedly endorsed by the National Council of Teachers of Mathematics.

The manipulative materials are designed to teach numeration, grouping, place value, area, perimeter, and the concepts and properties of addition, subtraction, multiplication, and division. They consist of units, tens, hundreds, and thousands blocks on a centimeter scale. Made of unstained maple, the unit blocks are 1 cm cubes, the tens blocks are 1 cm X 1 cm X 10 cm, the hundreds blocks are 1 cm X 10 cm X 10 cm, and the thousands blocks are 10 cm X 10 cm X 10 cm. A typical classroom assortment of Base 10 materials would include at least 100 units, 50 tens, 25 hundreds, and one thousands.

Purpose

As a result of the 1979 needs meeting in mathematics, a project is currently underway at APH to develop a set of introductory mathematics materials and activities for the primary grades. Also for APH in 1979, experts in mathematics education identified a core of critical concepts and learning objectives for these young students. Base 10 materials have had high utility in introducing and/or suplementing instruction in a number of these critical concepts with sighted pupils at the entry level in mathematics education.

It was the purpose of this project to determine the earliest age levels at which the Base 10 materials could be manipulated appropriately by young blind students and to develop guidelines for possible materials adaptation. Such information had value in determining whether or not to include Base 10 materials in the earliest levels of the Introductory Mathematics project, in subsequent levels of the same or a similar project in mathematics, or to have them offered separately as a tangible aid/software product. It was a sub-objective of this project to determine how performance on the manipulative tasks of the legibility items was related to age, sex, and visual orientation.

Subjects

Subject participation in this project occurred at two different levels. Initially, six legally blind subjects, two from each grade level K-2, were randomly selected from a residential school for the blind and a public school program for participation in piloting activities to provide information on ability to manipulate the materials, approximate length of testing time, and to suggest possible materials adaptation. Five of the subjects were males, four were print readers, three were between the ages of 5 and 6 years, and three were between the ages of 7 and 8 years. In the formal field evaluation, 48 legally blind pupils from three residential and two public school programs participated. All subjects were between the ages of 5 and 8 years old, and the group was evenly divided by sex, age group, and visual orientation. Furthermore, all subjects were enrolled in an academic educational program and had no additional handicap.

Materials

Materials consisted of a set of Number-Blox^{TM} commercially available from Creative Publications, Palo Alto, California (#30064). Manipulative tasks with the materials were performed on a smooth, table surface. Tasks involved in Prescribed Construction Variations were performed on white paper with gray-shaded patterns by visual subjects or on heavy cardboard with cutouts for the patterns by tactual subjects.

Procedure

In commercially available Base 10 program materials, specific activities are prerequisite to using the materials to introduce concepts of numeration, simple addition, and subtraction. These prerequisite activities occur in the following order: (1) free play with the materials; (2) identification of the four different blocks; (3) sorting activities; (4) duplication of prescribed constructions; and (5) equivalent matching. Representative tasks related to each of the last four prerequisite skill areas were identified that permit scoring on the basis of correct or incorrect performance. Items of the materials evaluation are included in the appendix.

Each subject participated in two sessions of the materials evaluation. In Session I, each subject successfully completing screening immediately proceeded into a training session composed of tasks identical to the formal testing items. Session II occurred on the following day and consisted of a repeat administration of the items used for training in Session I. Each session was 20-25 minutes in duration. The free play activities were limited to a five minute inspection of the materials at the beginning of the screening and Session I and were not subject to scoring.

Subjects successfully completing screening and familiarization actitities (screening instrument is included in the appendix) were grouped by age (5-6 years vs. 7-8 years), sex, and visual orientation (visual vs. tactual). The cell size was six and whenever a test site provided a surplus of qualified subjects for any given cell, those subjects participating in formal testing were randomly selected. Subjects who were braille readers with residual vision were required to wear sleep shades during the materials evaluation.

Legibility studies conducted as part of materials development at APH have typically established an 85% average of correct responses among a subject population as an acceptable indication of overall legibility of any particular aid. From the onset it was assumed that if average performance of any cell population in the areas of prescribed construction and/or equivalent matching fell below an 85% correct response criteria, some form of further materials adaptation would be in order.

Results

Of 50 subjects screened, two were dismissed for their inability to attend to tasks, indicating that 96% of the subjects invited to participate achieved a score of 100% on the screening instrument activities. In the evaluation, scores were computed for overall performance as well as for performance on subcategories of tasks related to equivalent matching, prescribed construction, and variations of prescribed construction. Generally, after two, 25 minute sessions with the unadapted Base 10 materials, subject performance failed to meet the pre-established legibility criteria of 85% correct responses (Table 6). A Kruskal-Wallis one-way analysis of variance by ranks applied to the overall performance data of the eight groups yielded

H=12.631, p < .10. The Mann-Whitney U Test revealed that overall performance differences were attributed mostly to visual orientation (U=443.5, p < .0007). Results of the Kurskal-Wallis one-way analysis of variance by ranks appear in Table 1, and results of the Mann-Whitney U Test appear in Tables 2-5.

In a simple task included to determine how many of the small unit cubes could be aligned correctly end-to-end, the overall average for subjects was 7.3 cubes (Table 7). Having the advantage of using an APH Work-Play Tray (#1-0375) for all tasks, subjects in the pilot test group averaged 8.8 units correctly aligned on the alignment task, and achieved an average total legibility score 15.5% higher than the field group (Table 8). Items in Prescribed Construction Variation required visual subjects to completely cover a printed pattern with unit blocks, while tactual subjects were instructed to completely fill in identical patterns cut into heavy cardboard with the units. The differences in performance scores on items of Prescribed Construction Variation between the tactual and visual field groups appear in Table 9. Data related to correctly counting blocks to match the number used in sample constructions is recorded in Table 10.

Table 1

Kruskal-Wallis One-Way Analysis of Variance by Ranks

Category	ΣT	H	df	p<
Equivalent Matching	2520	15.295	7	.05
Prescribed Construction	1464	13.010	7	.10
Prescribed Construction (Var.)	13524	9.714	7	.30
Overall Performance	798	12.631	7	.10

Table 2

Mann-Whitney *U* Test - Equivalent Matching

Category	ΣT	U	z	p
5-6 yrs. vs. 7-8 yrs.	219	381.0	1.9408	.0262
Visual vs. Tactual		443.0	3.2347	.0007
Male vs. Female		276.0	.2504	.4013

Table 3

Mann-Whitney *U* Test - Prescribed Construction

Category	ΣT	U	z	p
5-6 yrs. vs. 7-8 yrs.	122	319.0	.6435	.2611
Visual vs. Tactual		471.5	3.8090	.00007
Male vs. Female		292.0	.0830	.4681

Table 4

Mann-Whitney *U* Test - Prescribed Construction Variations

Category	ΣT	U	z	p
5-6 yrs. vs. 7-8 yrs.	1127	323.0	.7703	.2206
Visual vs. Tactual		203.5	1.8598	.0314
Male vs. Female		220.0	1.4967	.0668

Table 5

Mann-Whitney U Test - Overall Performance

Category	ΣT	U	z	p
5-6 yrs. vs. 7-8 yrs.	66	345.0	1.1796	.1190
Visual vs. Tactual		443.5	3.2180	.0007
Male vs. Female		275.0	.2690	.3936

Table 6

Legibility Scores
(Average % Correct)

Category*	5-6 yr. Male Tactual	5-6 yr. Male Visual	5-6 yr. Female Tactual	5-6 yr. Female Visual	7-8 yr. Male Tactual	7-8 yr. Male Visual	7-8 yr. Female Tactual	7-8 yr. Female Visual	Overall $\bar{X}$%
E.M.	41.1	53.3	24.4	50.0	38.9	66.7	47.8	61.1	47.9
P.C.	48.3	66.7	39.2	75.0	50.0	75.0	50.0	75.0	59.9
P.C.V.	85.0	25.0	50.0	40.0	85.0	65.0	50.0	40.0	55.0
Total	48.7	57.8	33.9	62.2	48.7	70.9	49.1	66.5	54.8

*Categories: E.M. (Equivalent Matching); P.C. (Prescribed Construction); P.C.V. (Prescribed Construction Variation)

Table 7

Alignment Scores
(Average No. in Alignment)

5-6 yr. Male Tactual	5-6 yr. Male Visual	5-6 yr. Female Tactual	5-6 yr. Female Visual	7-8 yr. Male Tactual	7-8 yr. Male Visual	7-8 yr. Female Tactual	7-8 yr. Female Visual	$\bar{X}$:5-6	$\bar{X}$:7-8	$\bar{X}$
6.7	8.2	5.3	8.3	7.0	8.2	6.0	8.7	7.1	7.5	7.3

Table 8

Average Legibility Scores
Pilot Group vs. Field Group
(Percent Responding Correctly)

Category	Pilot Group n=6	Field Group n=48
Equivalent Matching	67.8	47.9
Prescribed Construction	74.2	59.9
Prescribed Construction Variation	58.3	55.0
Total	70.3	54.8

Table 9

Performance Scores
Items of Prescribed Construction Variation
(Percentage of Subjects)

Students Performing:	Tactual	Visual
All Items Correctly	54.2	29.2
One Item Correctly	25.0	29.2
No Items Correctly	20.8	41.6
Total	100.0	100.0

Table 10

Average Legibility Scores
Items Related to Matching/Counting
(Percent Responding Correctly)

Item	Blocks	$\overline{X}$:5-6	$\overline{X}$:7-8	$\overline{X}$
1a	1000's	100.0	100.0	100.0
2a	100's	87.5	91.7	89.6
3a	10's	54.2	66.7	60.4
4a	1's	75.0	83.3	79.2
5a	100's	87.5	95.8	91.7
6a	10's	58.3	95.8	77.1
7a	1's	54.2	83.3	68.8

Discussion

In this study legally blind subjects, regardless of their young age, sex, or visual orientation, could easily identify the different Base 10 blocks as performance on the screening activities demonstrates. Although the data in Table 10 is more related to matching than counting, performance suggests that most students can count the various blocks easily up to a quantity of 10, that 7-8 year old subjects can count the blocks more accurately than those 5-6 years old, and that, generally, the larger the block, the fewer the number of errors committed in matching/counting activities.

On the average, visual subjects could align at least 8 unit blocks while tactual subjects aligned at least 5 blocks after only one other trial and without the aid of a work-play tray or a tens block to guide their efforts. This finding suggests that, with minimal training and experience with the materials, young students should be able to match the length of 10 units placed end-to-end with the length of a tens block and thereby gain the manipulative skills necessary to learn concretely concepts and operations of addition and subtraction involving sums and minuends greater than 10.

Other findings indicated that the performance of blind students between the ages of 5 and 8 years failed to achieve the pre-set standard for materials legibility within two short sessions with the unadapted Base 10 materials. In pilot testing, however, it was discovered that for subjects in the 5-8 year old age group many of the tasks could be performed correctly and easily on the first trial when done within an APH Work-Play Tray, since the high, right-angle corners of the tray acted to steady the blocks while comparisons with the next smaller sized block were being made.

Since the educational value of Base 10 materials is dependent on constructions and student examination of each construction, any materials adaptation must act to stabilize constructions. Indeed, poor stability was the key factor to the poor performance exhibited by the students in this study. Therefore, it is speculated that if a number of jigs and right-angle corners varying in height from 1-10 cm be developed which would fit into the APH Work-Play Tray, not only would performance on the items of legibility increase to acceptable levels, but adaptations with universal applications for utilizing the materials in the classroom could be identified that would make learning with the popular aid accessible to the young, visually handicapped student, as well. For example, an open, three-sided box 10 cm high, 10 cm wide, 20 cm long, and made of clear plexiglass could be constructed which would allow a student to stack 10 hundreds blocks beside a thousands block. Such a stabilizing aid would permit most students on their first trial to see that 10 hundreds are equivalent to 1 thousands. This one adaptation would also permit study of addition, subtraction, multiplication, and division involving sums, minuends, products, and dividends equal to 1000. Similarly, the data in Table 9 suggests that the patterns formed in relief for use by tactual subjects on the tasks of Prescribed Construction Variations added a degree of stability which enhanced their performance as a group over the visual subjects, and constituted an adaptation that would be useful for low vision students unable to utilize the printed patterns designed for normally sighted students.

APPENDIX

Screening/Familiarization

Student No. ________ Date of Birth____________ Grade______ Sex_______

School System_________ Visual or Tactual_______ Informed Consent________

"My name is____ and I'm from the American Printing House for the Blind. The American Printing House makes most of the books, paper, and learning materials you use in your classroom everyday. The reason I've come to your school today is to find some students who will help me look at some special wooden blocks we think students and teachers would like to have in their classroom. Before the American Printing House makes any new product to send all over the country, we like to have a few students use the materials to help us decide if they're really good materials. Will you help us? Fine (Record response above)."

(The following blocks are randomly heaped in a work/play tray before the student: 3-1000's, 12-100's, 12-10's, and 15-units.)

"I've placed some wooden blocks in front of you and I'd like you to take a minute to look at them (sleep shades must be worn by tactual students exhibiting here the use of residual vision). Notice that some of the blocks are very large and some are very small. Some blocks are long and skinny and some blocks are big and flat."

	SCORE (+/-)	COMMENTS
1) (Hand the student a 1000 block). "Show me another block from the pile that's just like this one."	______	
2) (Hand the student a 100 block). "Show me another block from the pile that's just like this one."	______	
3) (Hand the student a 10 block). "Show me another block from the pile that's just like this one."	______	
4) (Hand the student a unit block). "Show me another block from the pile that's just like this one."	______	
5) (Hand the student a 1000 block). "Give me all the blocks in this pile that are just like this one."	______	
6) (Return the 1000 blocks to the pile and hand the student a 100 block). "Give me all the blocks in this pile that are just like this one."	______	
7) (Return the 100 blocks to the pile and hand the student a 10 block). "Give me all the blocks in this pile that are just like this one."	______	

	SCORE	COMMENTS
8) (Return the 10 blocks to the pile and hand the student a unit block). "Give me all the blocks in this pile that are just like this one."	_____	

	SCORE	COMMENTS
9) Was the subject cooperative?	_____	
10) Did the subject attend to tasks?	_____	
11) Did the subject understand tasks?	_____	
12) Was the subject able to pick up each size block?	_____	

The criteria for determining if the student will be asked to continue through the remainder of the evaluation will be 100% correct responses to items 1-8 and four, "Yes," answers to questions 9-12. Should the subject meet this criteria , then continue with:

"Fine,______. You've been very helpful with this part. There are a few more activities I'd like you to try today and a few more I'd like for you to try tommorrow. Would you like to keep going today and again tommorrow?" (Record response to continue through the remainder of materials evaluation below). "Are you planning on being here tommorrow?" (Record below and dismiss subjects who will be absent more than one day.)

Informed consent _____

Planning to attend _____

For performance less than the above criteria, thank subject for participating and return student to class.

*****

Definitions of Terms Used in Testing

1) "Appropriate sides touching" - surface contact is at least 50%

2) "Construction essentially duplicated" - variance is no more than 1 cm

3) "Essentially. . . a straight line" - variance is no more than 5mm

Items for Sessions I & II

Session I: Continued below. (See following block supply note.)

Session II: (Thank subject for returning and give sleep shades to those who wore them during Session I. The supply of blocks contains: 8-1000's, 24-100's, 24-10's, and 30-units). "I'd like you to take a minute or two to look over these blocks again before I ask you a few more questions about them. They are the very same blocks you used yesterday."

****

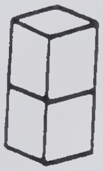
Prescribed Constructions — Score — Comments

1) (With the supply of 1000 blocks, replicate the construction shown below). "Look at what I've made with these blocks. (Place hands on the construction and allow student to examine). I'd like you to take some of the blocks in front of you and make this same thing."

   a) Were the correct numbers of blocks used? ______

   b) Are the appropriate sides of the blocks touching? ______

   c) Was the prescribed construction essentially duplicated? ______



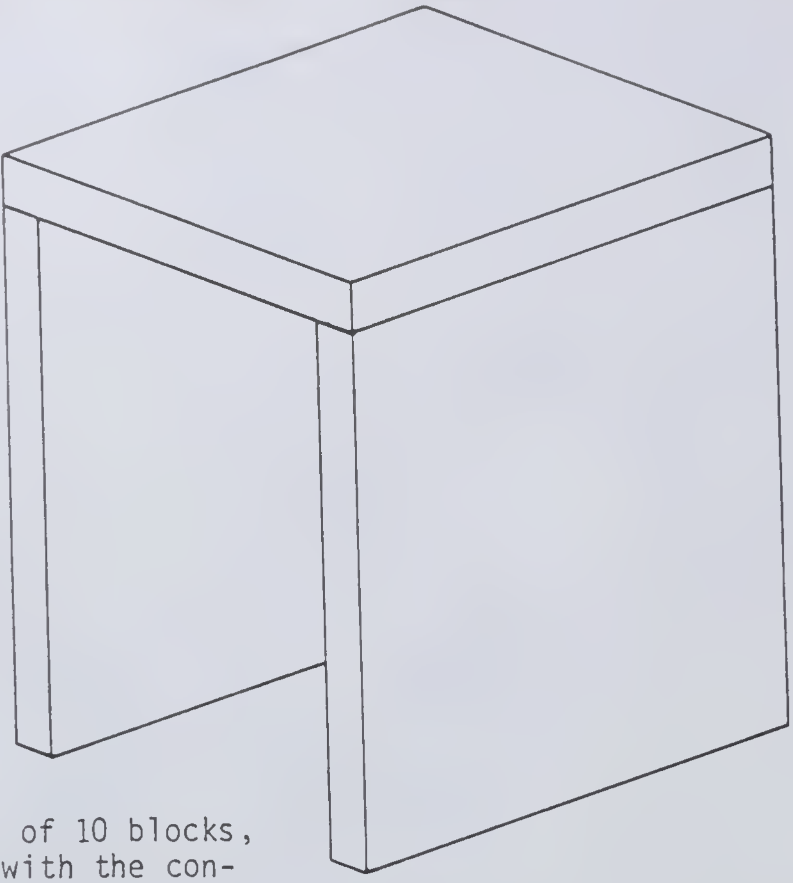
(Scale: 1cm = 10cm)

2) (With the supply of 100 blocks, repeat #1 above with the construction shown below.)

   a) Were the correct numbers of blocks used? ______

   b) Are the appropriate sides of the blocks touching? ______

   c) Was the prescribed construction essentially duplicated? ______

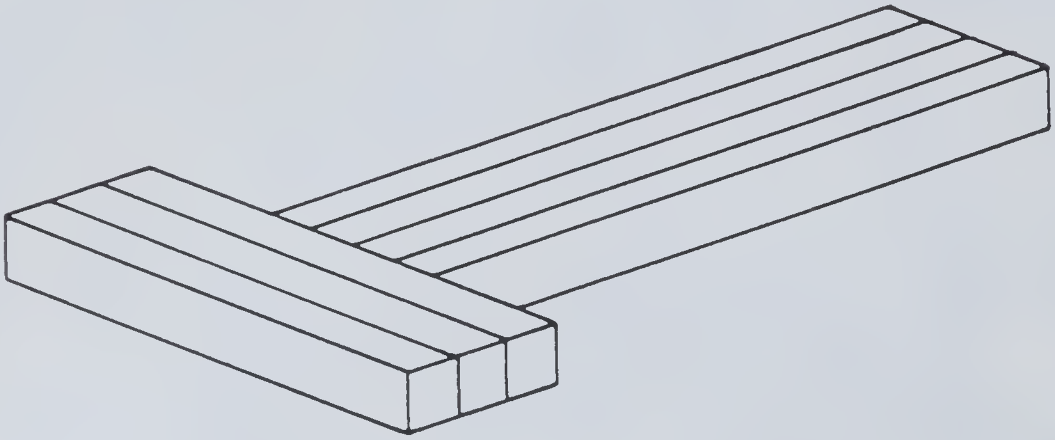


3) (With the supply of 10 blocks, repeat #1 above with the construction shown below.)

a) Were the correct numbers of blocks used? ______

b) Are the appropriate sides of the blocks touching? ______

c) Was the prescribed construction essentially duplicated? ______



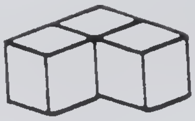
Prescribed Constructions (continued) Score Comments

4) (With the supply of unit blocks, repeat #1 above with the construction shown below.)

   a) Were the correct numbers of blocks used? ______

   b) Are the appropriate sides of the blocks touching? ______

   c) Was the prescribed construction essentially duplicated? ______



* * *

Equivalent Matching

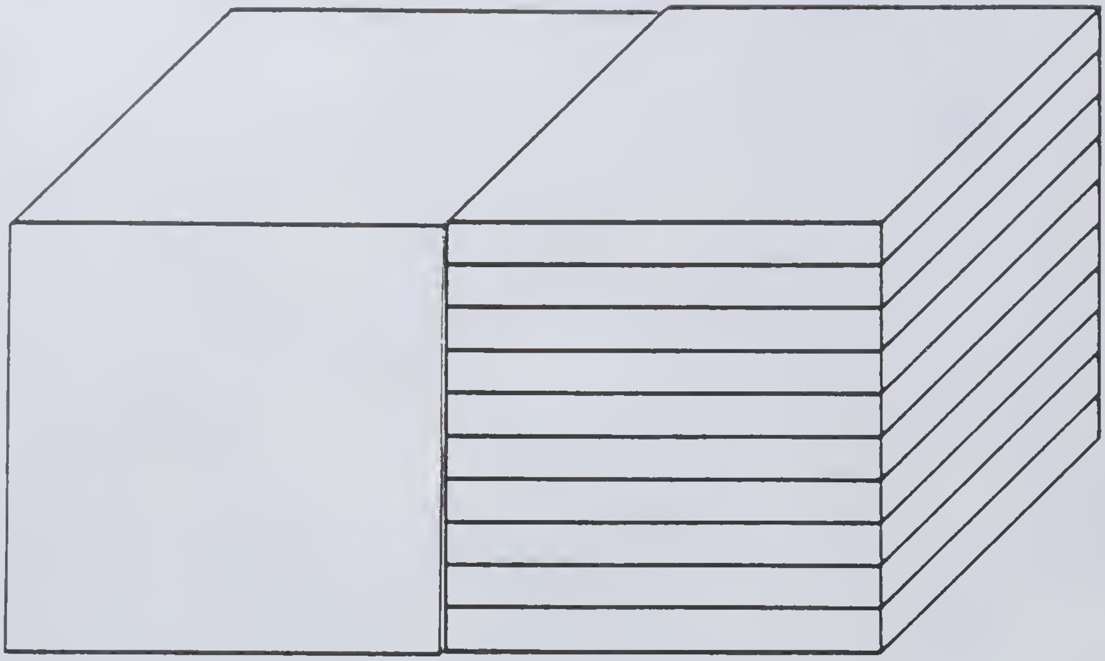
5) (With the supply of 1000 and 100 blocks, replicate the construction shown below). "Look at what I've made with these blocks. (Place hands on the construction and allow student to examine). I'd like you to take some of the blocks in front of you and make this same thing."

   a) Are the correct numbers of blocks used? ______

   b) Are the appropriate sides of the blocks touching? ______

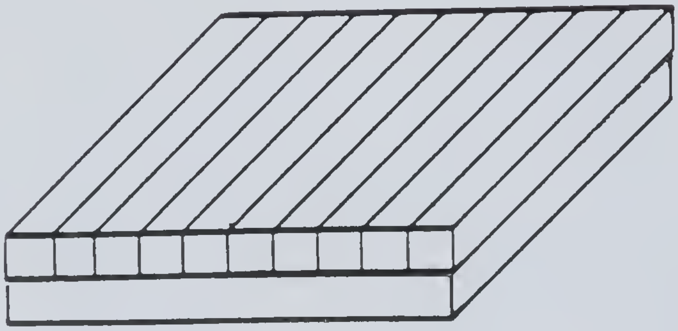
   c) Essentially are the blocks in a straight line? ______

Equivalent Matching (continued) Score Comments



6) (With the supply of 100 and 10 blocks, repeat #5 above with the construction shown below.)

- a) Are the correct numbers of blocks used? ______
- b) Are the appropriate sides of the blocks touching? ______
- c) Essentially are the blocks in a straight line? ______



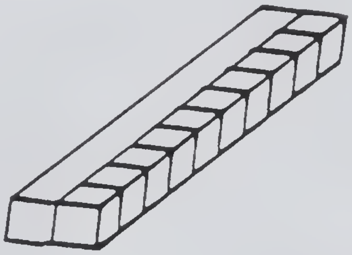
Equivalent Matching (continued) Score Comments

7) (With the supply of 10 and unit blocks, repeat #5 above with the construction shown below.)

   a) Are the correct numbers of blocks used? ______

   b) Are the appropriate sides of the blocks touching? ______

   c) Essentially are the blocks in a straight line? ______



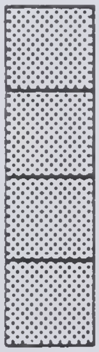
****

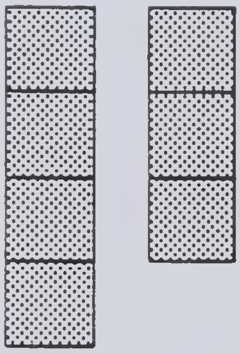
Prescribed Construction Variations

8) (For items #8 & #9, visual subjects will use print copies of the patterns and tactual subjects will use a cardboard jig. Use the supply of unit blocks.) "I have covered the dark drawing with the small blocks. I'd like you to cover your drawing in just the same way."

   a) Were the shaded areas completely covered with the correct number of blocks? ______

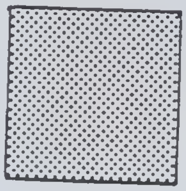
Prescribed Construction Variations Score Comments

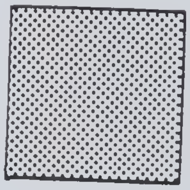




9) (Repeat #8 with the figure below.)

a) Was the shaded area completely covered with the correct number of blocks? ______





Equivalent Matching Variation	Score (No. 1-10)	Comments
10) (Allow student to observe as you line up 10 unit blocks along a 10 block. When all 10 unit blocks are aligned, remove the 10 block guide and let student inspect the 10 unit blocks in a straight line with no gaps between blocks.) "I used a long, skinny block to help me make a straight line with the little blocks, but I want you to see how many little blocks you can put in a straight line without using the long, skinny block. (Give student up to 10 unit blocks one at a time and record the number of blocks properly aligned with sides touching.)	______